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Hybrid static output feedback stabilization of second-order linear time-invariant systems

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Abstract

For second-order linear time-invariant (LTI) systems which are not stabilizable via a single static output feedback, we study the open problem whether there exists a finite-state hybrid static output feedback to asymptotically stabilize the system. We show that the answer to this question is affirmative for a class of second-order LTI systems, by constructing a 2-state static output feedback incorporated with a conic switching law. © 2002 Elsevier Science Inc. All rights reserved.

Keywords: Second-order LTI system; Hybrid static output feedback; Stabilization; Conic switching law

1. Introduction

Consider the linear time-invariant (LTI) control system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1.1}$$

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where $x \in \mathbb{R}^n,\ u \in \mathbb{R}^m,\ y \in \mathbb{R}^p$ are the state, the control input and the measurement output, respectively, and A, B, C are constant matrices of suitable dimension. We assume that the triple (A, B, C) is controllable and observable.

The stabilization problem of system (1.1) via a single static output feedback has been studied exhaustively; see the survey paper [16] and the references cited therein. However, when system (1.1) is not stabilizable via a single static output feedback, it is necessary to consider a hybrid stabilization method, where a family of static output feedbacks should be included. In the following, we present a motivation example, which was also discussed in [1,7,8].

Example 1. Consider the harmonic oscillator model with position measurement described by the following equations:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \\ y &= [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \end{aligned} \tag{1.2}$$

Although the above system is both controllable and observable, it cannot be stabilized by a single static output feedback [1]; however, it is stabilizable by a hybrid static output feedback [1,7]. By letting $u = -y$ and $u = \frac{1}{2}y$, we obtain the following systems, respectively,

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{1.3}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{1.4}$$

Define $V(x) \stackrel{\triangle}{=} x_1^2 + x_2^2$. If system (1.3) is active in the first and third quadrants, while system (1.4) is active in the second and fourth quadrants, we will have $\dot{V} < 0$ whenever $x_1x_2 \neq 0$, which implies that the entire switched system is asymptotically (and hence, for linear systems, exponentially) stable by LaSalle's Principle (e.g., [15]).

We observe from the above example that when system (1.1) is not stabilizable by a single static output feedback, it is possible to find a hybrid static output feedback, which is composed of a family of static output feedback controllers and a switching strategy determining which controller should be activated at every instant. Motivated by the above observation, we formulate the control problem in this paper as follows.

Hybrid static output feedback stabilization problem. For system (1.1), find a finite collection of static output feedback controllers $u = K_i y,\ K_i \in \mathbb{R}^{m\times p},\ i = 1, \ldots, N$, and a piecewise constant switching signal $\sigma(t, y) : [0, \infty) \times \mathbb{R}^p \to \{1, \ldots, N\}$ such that the switched system

$$\dot{x}(t) = \left[A + BK_{\sigma(t,y)}C\right]x(t) \tag{1.5}$$

is asymptotically stable.

The piecewise constant controller $u = K_{\sigma(t,y)}y$ is called an N-state (hybrid) static output feedback controller in this paper. To rule out trivial cases, we always assume that there exists no matrix K such that $A + BKC$ is Hurwitz stable.

We now review several related existing results. In [9], it has been shown that if system (1.1) is controllable and observable, then it admits a stabilizing hybrid output feedback that uses a countable number of discrete states. In [1] as well as [7], the question is posed whether it is possible to stabilize system (1.1) by using a hybrid static output feedback with only a finite number of discrete states. Several specific examples that are included in [1] suggest that the answer to this question may be affirmative, and a sufficient condition based on multiple Lyapunov functions is derived in [7]. However, the problem is far from resolved, and the problem remains open, as pointed out also in [7]. The present situation is partially due to the sparsity of stability results for switched systems that can be used to deal with stabilization problems of this kind. Stabilization results come usually after the discovery of suitable stability results, but the existing general stability results (e.g., [2,3,6,8,10–14,17,20–22]) for switched systems do not seem to be applicable to stabilization problems, either because they are too specific to the systems that are addressed, or because they are not computationally tractable. For example, we are enlightened by Example 1 that a common Lyapunov-like function $V(x)$ associated with an approapriate partition of the $\mathbb{R}^n$ plane should be a useful approach to the hybrid control problem, but up to now we have obtained desirable results only for second-order switched systems in several cases [5]. For recent progress in the stability analysis and design of switched systems, refer to the survey papers [3] and [8].

In this paper, we focus on second-order single-input-single-output (SISO) systems, i.e., $n = 2$, $m = p = 1$ in system (1.1). Since a nonsingular linear transformation does not change the stabilization property, we consider system (1.1) in the following controllable canonical form:

$$A = \begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0], \tag{1.6}$$

where $a, b \in \mathbb{R}$, and we assume throughout this paper that $a \geqslant 0$. It is easy to see that the above system is not stabilizable by a single static output feedback. Therefore, though system (1.1) with (1.6) does not cover all controllable canonical forms of second-order LTI systems, it is an interesting class. For this class of systems, we aim to present a complete solution to the hybrid static output feedback stabilization problem. The solution is composed of two important parts. First, we prove that the hybrid control problem in hand for system (1.1) with (1.6) is solvable. Secondly, we design such a hybrid static output feedback with the number of necessary static output feedback controllers being two. To conclude, we construct a 2-state static output feedback to asymptotically stabilize system (1.1) with (1.6).

The rest of this paper is organized as follows. Since our proposed hybrid static output feedback is incorporated with the so-called conic switching law, we summarize in Section 2 the conic switching law proposed in [18,19] for switched systems consisting of two second-order subsystems. Section 3 states the main theorem and describes how to construct the hybrid controller. A numerical example demonstrates the effectiveness of the result. Finally, Section 4 gives some concluding remarks.

2. Conic switching law

In the interests of completeness and clarity, we summarize in the present section the conic switching law proposed in [18,19] for switched systems consisting of two second-order subsystems. For the benefit of this paper, we concentrate on the case where the two subsystems have unstable foci and are of the same clockwise direction. Here, as in [19], we say that a system is of clockwise direction if starting from any nonzero initial condition in the phase plane its trajectory is a spiral around the origin in the clockwise direction.

Consider the switched system described by two equations of the form

$$\dot{x}(t) = A_i x(t), \quad i = 1, 2, \tag{2.1}$$

where the two subsystems have unstable foci and are of clockwise direction. Let $x = [x_1 \;\; x_2]^{\mathrm{T}}$ and denote $A_1 x = [a_1 \;\; a_2]^{\mathrm{T}}$, $A_2 x = [a_3 \;\; a_4]^{\mathrm{T}}$. Although a_i's depend on x, we omit the argument x since no confusion will arise.

As in [18,19], we define for $i = 1, 2$ the following regions:

$$E_{is} = \left\{x \mid x^{\mathrm{T}} A_i x \leqslant 0\right\}, \quad E_{iu} = \left\{x \mid x^{\mathrm{T}} A_i x \geqslant 0\right\}, \tag{2.2}$$

and the following conic regions as a partition of the entire $\mathbb{R}^2$ plane:

$$\begin{aligned}
\Omega_1 &= E_{1s} \cap E_{2u}, \\
\Omega_2 &= E_{1u} \cap E_{2s}, \\
\Omega_3 &= E_{1s} \cap E_{2s} \cap \left\{x | a_2 a_3 - a_1 a_4 \leqslant 0\right\}, \\
\Omega_4 &= E_{1s} \cap E_{2s} \cap \left\{x | a_2 a_3 - a_1 a_4 \geqslant 0\right\}, \\
\Omega_5 &= E_{1u} \cap E_{2u} \cap \left\{x | a_2 a_3 - a_1 a_4 \leqslant 0\right\}, \\
\Omega_6 &= E_{1u} \cap E_{2u} \cap \left\{x | a_2 a_3 - a_1 a_4 \geqslant 0\right\}.
\end{aligned} \tag{2.3}$$

By associating subsystem 1 with $\Omega_1, \Omega_3, \Omega_5$ and subsystem 2 with $\Omega_2, \Omega_4, \Omega_6$, we obtain the conic switching law proposed in [18,19]. The following result concerns the stabilizability of the switched system (2.1).

Lemma 1. *Let l_1 be a ray that passes through the origin and let $x_0 \neq 0$ be on l_1. Let x^* be the point on l_1 where the trajectory intersects l_1 for the first time after leaving x_0, when the switched system evolves according to the conic switching law. Then, the switched system* (2.1) *is asymptotically stabilizable by the conic switching law if and only if $\|x^*\|_2 < \|x_0\|_2$.*

In the next section, we will design two static output feedback controllers for system (1.1) with (1.6) so that the two resulting closed-loop systems are of clockwise direction and the entire switched system is stabilizable by the conic switching law in the sense of Lemma 1.

3. Hybrid static output feedback

We are now in a position to state and prove the main result of the present paper.

Theorem 1. *System* (1.1) *with* (1.6) *is stabilizable via a* 2*-state hybrid static output feedback.*

Proof. For system (1.1) with (1.6), the coefficient matrix of the closed-loop system with a static output feedback $u = ky$ is given by

$$A_{b+k} = A + kBC = \begin{bmatrix} 0 & 1 \\ b+k & a \end{bmatrix}, \tag{3.1}$$

where we denote

$$A_\tau \triangleq \begin{bmatrix} 0 & 1 \\ \tau & a \end{bmatrix}.$$

Equivalently, we may work directly with A_τ instead of A_{k+b}. Since the characteristic equation for the matrix A_τ is $z^2 - az - \tau = 0$, if we choose τ such that

$$\Delta = a^2 + 4\tau < 0, \tag{3.2}$$

then A_τ's eigenvalues are expressed as

$$z_\pm = \alpha \pm \beta \mathrm{i} \triangleq \frac{a}{2} \pm \frac{\sqrt{|\Delta|}}{2}\mathrm{i}. \tag{3.3}$$

We also know that when $\Delta < 0$ the trajectories of $\dot{x} = A_\tau x$ proceed in clockwise direction.

In the following, we assume that $a > 0$ and $\Delta < 0$ for each τ that we will choose. ($a = 0$ may be viewed as a limiting case of $a > 0$ as a approaches 0. We will present a direct analysis for this case in Remark 1.) From now on, we consider a 2-state static output feedback for the system. Letting

$$
\begin{aligned}
A_{\tau_1}x &= \begin{bmatrix} 0 & 1 \\ \tau_1 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \\
A_{\tau_2}x &= \begin{bmatrix} 0 & 1 \\ \tau_2 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \triangleq \begin{bmatrix} a_3 \\ a_4 \end{bmatrix},
\end{aligned}
\tag{3.4}
$$

we obtain that

$$
\begin{aligned}
&a_2a_3 - a_1a_4 = (\tau_1 - \tau_2)x_1x_2, \\
&x^{\mathrm{T}}A_{\tau_1}x = x_2\big[ax_2 + (1+\tau_1)x_1\big], \\
&x^{\mathrm{T}}A_{\tau_2}x = x_2\big[ax_2 + (1+\tau_2)x_1\big].
\end{aligned}
\tag{3.5}
$$

If we further assume that

$$
\tau_1 < \tau_2 < -1, \tag{3.6}
$$

then according to the discussion in Section 2, the conic switching law can be identified as follows: choose subsystem A_{τ_1} if $x_1x_2 > 0$ and choose subsystem A_{τ_2} whenever $x_1x_2 < 0$. (On the boundary $x_1x_2 = 0$, we can simply choose A_{τ_1}. This is of no importance in the present case since the directions of both subsystems are identically clockwise.) The partition of the $\mathbb{R}^2$ plane is depicted in Fig. 1.

Invoking the conic switching law, we start from a point $(\omega_0, 0)$ on the x_1-axis, and follow subsystem A_{τ_2} clockwisely until the trajectory reaches the x_2-axis at $(0, \omega_1)$. Next, we follow subsystem A_{τ_1} in clockwise direction until the trajectory reaches the x_1-axis again at $(\omega_2, 0)$. By Lemma 1, since the switching law is identically repeated

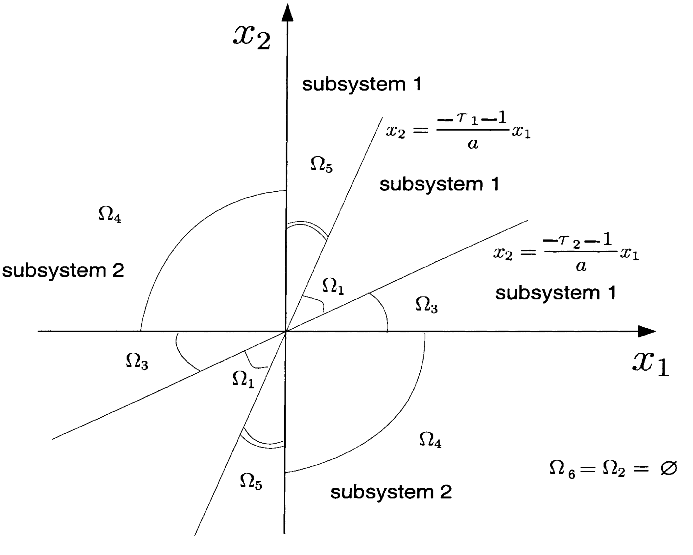

Fig. 1. Partition of the $\mathbb{R}^2$ plane in the proof of Theorem 1.

on the other half plane, we require $-1 < \omega_2/\omega_0 < 0$ so that the switched system is asymptotically stable.

Writing the differential equation $\dot{x} = A_{\tau_2}x$ as $\ddot{x}_1 - a\dot{x}_1 - \tau_2 x_1 = 0$ with the initial value $(x_1, \dot{x}_1) = (\omega_0, 0)$, we obtain

$$\begin{aligned} x_1(t) &= \omega_0 e^{\alpha_2 t}\left[\cos(\beta_2 t) - \frac{\alpha_2}{\beta_2}\sin(\beta_2 t)\right], \\ x_2(t) &= -\frac{1}{\beta_2}(\alpha_2^2 + \beta_2^2)\omega_0 e^{\alpha_2 t}\sin(\beta_2 t). \end{aligned} \tag{3.7}$$

Similarly, writing the differential equation $\dot{x} = A_{\tau_1}x$ as $\ddot{x}_1 - a\dot{x}_1 - \tau_1 x_1 = 0$ with the initial value $(x_1, \dot{x}_1) = (0, \omega_1)$, we have

$$\begin{aligned} x_1(t) &= \frac{\omega_1}{\beta_1} e^{\alpha_1 t}\sin(\beta_1 t), \\ x_2(t) &= \frac{\omega_1}{\beta_1} e^{\alpha_1 t}\left[\alpha_1 \sin(\beta_1 t) + \beta_1 \cos(\beta_1 t)\right]. \end{aligned} \tag{3.8}$$

In the above, α_i and β_i $(i = 1, 2)$ are given in (3.3) with respect to τ_i.

Let t_2 denote the time expired by following subsystem A_{τ_2} from $(\omega_0, 0)$ to $(0, \omega_1)$, and let t_1 denote the time expired by following subsystem A_{τ_1} from $(0, \omega_1)$ to $(\omega_2, 0)$. Solving $x_1(t_2) = 0$ in (3.7) and $x_2(t_1) = 0$ in (3.8), respectively, we obtain that

$$t_2 = \frac{1}{\beta_2}\tan^{-1}\frac{\beta_2}{\alpha_2}, \quad t_1 = \frac{1}{\beta_1}\left(\pi - \tan^{-1}\frac{\beta_1}{\alpha_1}\right), \tag{3.9}$$

and

$$\begin{aligned} \omega_1 &= -\frac{1}{\beta_2}(\alpha_2^2 + \beta_2^2)\omega_0 e^{\alpha_2 t_2}\sin(\beta_2 t_2), \\ \omega_2 &= \frac{\omega_1}{\beta_1} e^{\alpha_1 t_1}\sin(\beta_1 t_1). \end{aligned} \tag{3.10}$$

Hence, what we require for asymptotical stability of the switched system is

$$\frac{1}{\beta_1} e^{\alpha_1 t_1}\sin(\beta_1 t_1)\cdot\frac{1}{\beta_2}(\alpha_2^2 + \beta_2^2) e^{\alpha_2 t_2}\sin(\beta_2 t_2) < 1. \tag{3.11}$$

Using

$$\sin(\beta_i t_i) = \frac{\beta_i}{\sqrt{\alpha_i^2 + \beta_i^2}} \quad \text{and} \quad \alpha_i = a/2,$$

we rewrite (3.11) as

$$\exp\left[a\left(\frac{1}{\beta_2}\tan^{-1}\frac{2\beta_2}{a} + \frac{1}{\beta_1}\left(\pi - \tan^{-1}\frac{2\beta_1}{a}\right)\right)\right] < \frac{a^2 + 4\beta_1^2}{a^2 + 4\beta_2^2}. \tag{3.12}$$

From the facts that

$$\left|\frac{1}{\beta_2}\tan^{-1}\frac{2\beta_2}{a}\right| \leqslant \frac{2}{a}, \quad \pi - \tan^{-1}\frac{2\beta_1}{a} < \pi,$$

and

$$\frac{a^2+4\beta_1^2}{a^2+4\beta_2^2} \geqslant \frac{4\beta_1^2}{(a+2\beta_2)^2},$$

a less restrictive sufficient condition for (3.12) can be derived as

$$\beta_1 > \left(\frac{a}{2}+\beta_2\right)\mathrm{e}^{1+a\pi/2\beta_1}. \tag{3.13}$$

In view of the discussion thus far, we should choose the constants k_1 and k_2 so that $\tau_i = b + k_i$ satisfies (3.2), (3.6) and (3.13). It is not difficult to show that such k_1 and k_2 always exist. Therefore, we conclude that system (1.1) with (1.6) is always stabilizable via a 2-state hybrid static output feedback. □

In the proof of Theorem 1, we focused on establishing stability of the entire switched system, without mentioning the initial point. Recalling that only the output $y = [1\ \ 0]x = x_1$ can be used according to our problem formulation, we see that our *switching strategy* consists of the following two steps:

Step 1. Starting from any initial point which is not necessarily located on the x_1-axis or x_2-axis, we activate any one of the two subsystems (both are of clockwise direction) until the trajectory reaches the x_2-axis. This is possible because $x_1 = 0$ is observable.

Step 2. Activate subsystem A_{τ_1} in the first and the third quadrants (i.e., $x_1x_2 > 0$) and subsystem A_{τ_2} in the second and the fourth quadrants (i.e., $x_1x_2 < 0$) alternatively, by using the coefficient matrix A_{τ_i}'s to compute exactly the time expired by certain subsystem from one axis to another axis, as was done in (3.9).

Remark 1. For the case of $a = 0$, we only need to choose τ_1, τ_2 to satisfy (3.2), (3.6) and $\beta_1 > \beta_2$ (by (3.12)), and thus it is an easy matter to know that two feedback gains k_1 and k_2 always exist so that the switched system is asymptotically stable. In the following, we provide further details concerning the feedback gains for the present case.

When $b < 0$, we can choose the feedback gains k_1, k_2 as small as we want. In fact, the only requirement on k_1, k_2 is $k_1 + b < k_2 + b < 0$. We implement the following switching law: choose subsystem A_{τ_1} when $x_1x_2 > 0$ and subsystem A_{τ_2} when $x_1x_2 < 0$. Then, we have for the switched system a common Lyapunov-like function

$$V(x) = \frac{|k_1+b|+|k_2+b|}{4}x_1^2 + \frac{1}{2}x_2^2$$

which satisfies

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \begin{cases} \frac{1}{2}\big(|k_2+b|-|k_1+b|\big)x_1x_2 < 0, & x_1x_2 > 0, \\ \frac{1}{2}\big(|k_1+b|-|k_2+b|\big)x_1x_2 < 0, & x_1x_2 < 0. \end{cases} \tag{3.14}$$

Therefore, the system is stabilizable via 2-state hybrid static output feedback with arbitrarily small gains. Analogous arguments apply to the case of $b = 0$ as well.

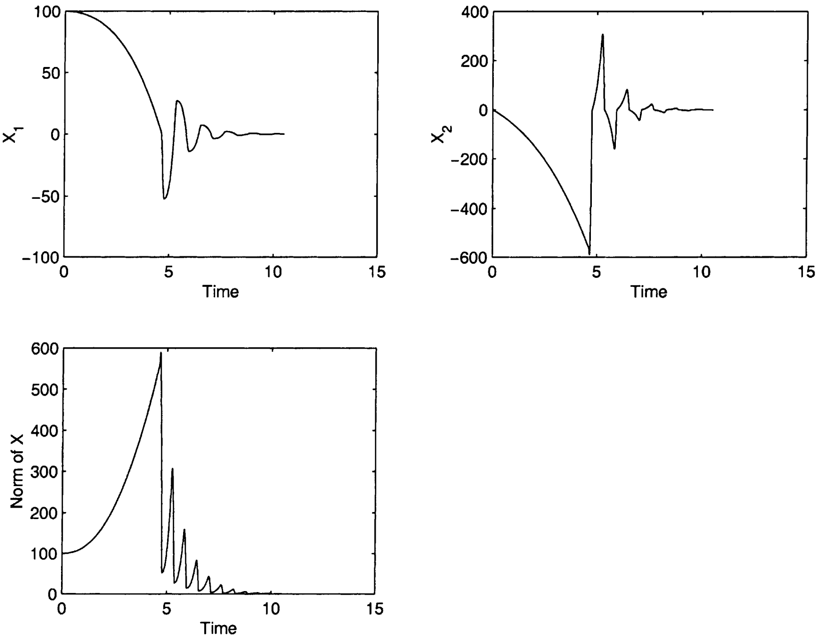

Fig. 2. Convergence of x_1, x_2 and $\|x\|$ in Example 2.

However, when $b > 0$, no such small gain property exists, since if we choose k_1, k_2 sufficiently small, we will always have $\mathrm{d}(x_1 x_2)/\mathrm{d}t > 0$, which excludes the possibility of small gain 2-state hybrid static output feedback. For such a case, we have to refer to the approach given in the proof of Theorem 1. Similar arguments on small gain property also apply for the other noncritical cases (when $a > 0$). We conclude that only in the case of $a = 0$ and $b \leqslant 0$ we can choose the 2-state hybrid static output feedback with arbitrary small gains.

Remark 2. Theorem 2 also gives a partial answer for second-order LTI systems to the following question that was raised in [7]: what advantage can be gained by switching between more than two subsystems? The answer is "*none*" in the hybrid stabilization problem addressed herein. However, in general, when the number of subsystems in a switched system increases, the answer should be affirmative. For instance, Example 3.2 in [4] shows that although any switching strategy between any two subsystems described there cannot achieve global stability, there exists a switching strategy that globally stabilizes the entire switched system by activating three subsystems.

We now give an example to demonstrate the applicability Theorem 1.

Example 2. We consider the stabilization problem via finite-state hybrid static output feedback for the following second-order system:

$$A = \begin{bmatrix} 0 & 1 \\ -13 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{3.15}$$

After some simple calculations, conditions (3.2), (3.6) and (3.13) in the proof of Theorem 1 are obtained as

$$\begin{aligned} & k_1 < k_2 < 9, \\ & 9 - k_1 > \left[13 - k_2 + 4\sqrt{9 - k_2}\,\right] \mathrm{e}^{2+4\pi/\sqrt{9-k_1}}. \end{aligned} \tag{3.16}$$

Thus, we can easily verify that the pair $(k_1, k_2) = (-180, 8)$ satisfies the required conditions. Fig. 2 depicts the convergence of x_1, x_2 and the norm of x of the switched system with the initial value $x_0 = [100 \;\; 0]^{\mathrm{T}}$ under the conic switching law: choose subsystem A_{τ_1} (thus the controller $u = k_1 y$) if $x_1 x_2 > 0$ and subsystem A_{τ_2} (thus the controller $u = k_2 y$) whenever $x_1 x_2 < 0$.

4. Concluding remarks

In the present paper, we used some recent results on conic switching law to study the open problem whether there exists a finite-state hybrid static output feedback to asymptotically stabilize second-order linear time-invariant systems. For a class of such systems, we showed that the system is stabilizable via a 2-state static output feedback incorporated with an appropriate conic switching law. Though we did not cover all cases, we suggest that the proposed hybrid controller design is practical for general second-order LTI systems.

We conclude by noting that since the hybrid controller design in this paper is based on conic switching law which was originally proposed for second-order switched systems, it is quite difficult to extend Theorem 1 to the case of general high-dimensional linear systems. For example, even for the three-dimensional system with the simple form

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0 \quad 0], \tag{4.1}$$

it is not easy to prove or disprove the existence of a stabilizing hybrid static output feedback controller. This remains an open problem for further research.

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